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# Supersymmetric webs of D3/D5-branes in supergravity

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Abstract: We study webs of D3- and D5-branes in type-IIB supergravity. These webs preserve at least 8 supercharges. By solving the Killing spinor equations we determine the form of supergravity solutions for the system. We then turn to the sub-class of the intersecting D3/D5 brane system and elucidate some of its features.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence.



 

## Contents



## 1. Introduction

In this paper we examine webs of D3-branes and D5-branes from the point of view of supergravity. There are at least three motivations for studying this system. The first motivation is to remedy our lack of understanding of localized intersections of branes [1] and more generally of branes ending on branes [2] in supergravity. Examples of such systems may help in unravelling some of their features. The second originates in Hanany and Witten's [3] construction of 3-dimensional N=4 gauge theories using webs of NS5-branes and D3-branes, the S-dual of which are the webs studied here. A Maldacena type limit [4] of the supergravity solution of such brane configurations should yield supergravity duals of these theories. These supergravity duals may help elucidate non-perturbative aspects of the gauge theories. The third motivation comes from the recent work of Karch and Randall [5] on constructing localized gravity models on branes and more generally branes in asymptotically AdS geometries. In these models one views the D5-branes as domain walls in the near-horizon $AdS_5$ geometry of the D3-branes. Such systems are of interest both because of their cosmological implications and also for their intrinsic interest as string theory backgrounds.

The main results of this paper are as follows. We develop a supersymmetric ansatz for the fields in type-IIB supergravity capable of accomodating webs of D3/D5-branes. The metric and form fields have certain interesting features which we elucidate. Using the ansatz we study in closer detail the case of the intersecting D3/D5-brane system. We present a simple but non-linear set of equations for this sub-case. We present solutions to this system of equations in the smeared D5-brane approximation in certain asymptotic regimes.

The structure of the paper is as follows. In section 2 we describe the brane set-up and the supersymmetry preservation conditions in the probe approximation. In section 3 we propose an ansatz for the metric and examine the Killing spinor equations in type-IIB supergravity with appropriate fields turned on and determine a class of supersymmetric field configurations. Then in section 4 we restrict ourselves to the sub-class of intersecting branes and remark on the structure of supergravity solutions for these configurations. We set up the Bianchi and source equations and solve them in the smeared approximation in certain asymptotic regimes. In section 5 we end with some concluding remarks.

## 2. Webs of D3/D5-branes

Consider a web of D3-branes and D5-branes where the D3-branes are along the $x^1, x^2, x^6$ directions while the D5-branes are along the $x^1, x^2, x^3, x^4, x^5$ directions. The D5-branes are located at $x^7 = x^8 = x^9 = 0$ and separated in the $x^6$ direction. The D3-branes, on the other hand, are located at $x^7 = x^8 = x^9 = 0$ but can be seperated in the $x^3, x^4, x^5$ directions. The D3-branes end on the D5-branes and stretch in the $x^6$ direction between adjacent D5-branes. When the D3-branes are aligned to form infinite D3-branes they can be lifted off the D5-branes. This important sub-case of infinite D3-branes will be referred to as the intersecting branes case.

The S-dual of the above set-up is known as the Hanany-Witten system [3], it was used by the authors to construct $N = 4$ Yang-Mills in string theory. We review some features below.

The S-dual of the D3/D5-brane system is the system described above but with the D5-branes replaced by NS5-branes. The D3-branes have four dimensional worldvolume theories truncated in the $x^6$ direction by the NS5-branes. Thus macroscopically, or at low energies, the D3-brane worldvolume theories are effectively a product of three dimensional gauge theories with gauge groups $\mathrm{SU}(n_i)$, where $n_i$ is the number of D3-branes stretched between the ith and (i+1)th NS5-branes. The coupling constants of the gauge theories are $1/g^2_{YM,i} = L_i/g_s$ where $L_i$ is the coordinate distance in $x^6$ between the ith and (i+1)th NS5-brane and $g_s$ is the string coupling constant. The $L_i$ specify UV cut-offs in real space (since probing distances smaller than the cut-off will allow one to probe the 4-th dimension). The limit in which we take the $L_i \to 0$, is the limit in which the Yang-Mills coupling becomes infinitely strong. This limit corresponds to an infrared limit of the quatnum field theory.

The positions of the D3-branes in the $x^3, x^4, x^5$ directions parameterize the Coulomb branch of the gauge theory living on the D3-branes, while also acting as bare masses of the fundamental matter multiplets of the theories living on adjacent intervals. Finally, the positions of the NS5-branes in the overall transverse direction (fixed to be at the origin in everything that follows) parameterize Fayet-Illiopoulis terms.

In the remainder of the paper we will be studying the S-dual of the D3/NS5-brane system described above. S-duality in the field theory acts by dualizing the gauge-field into a periodic scalar and inverting the Yang-Mills coupling constant. On the supergravity side S-duality replaces the NS5-branes with D5-branes and inverts the string coupling.

One can view the above brane set-up in a probe approximation in which the influence of the branes on the bulk geometry is neglected. This is a useful approximation when trying to determine the amount of preserved supersymmetry. Essentially one introduces open strings with appropriate boundary conditions for the above brane configuration in flat Minkowski space and determines the amount of supersymmetry that is preserved [6, 7]. For Dp-branes the supersymmetry preservation condition is [6, 7]:

$$\epsilon_L = \hat{\Gamma}_{012..p}\epsilon_R \tag{2.1}$$

where the $\hat{\Gamma}$ are flat space $\Gamma$-matrices, and $\epsilon_L, \epsilon_R$ are worldsheet left- and right-handed space-time spinors acting as supersymmetry variation parameters. In the case at hand the amount of space-time supersymmetry preserved was determined in [3], we repeat the calculation here for completeness. The variation parameters satisfy:

$$\begin{aligned} \epsilon_L &= \hat{\Gamma}_{0126}\epsilon_R \\ \epsilon_L &= \hat{\Gamma}_{012345}\epsilon_R \,. \end{aligned} \tag{2.2}$$

The spinors $\epsilon_L, \epsilon_R$ are Majorana-Weyl fermions satisfying:

$$\begin{aligned} \epsilon_L &= \hat{\Gamma}_{0123456789}\epsilon_L \\ \epsilon_R &= \hat{\Gamma}_{0123456789}\epsilon_R \,. \end{aligned} \tag{2.3}$$

The above constraints are satisfied by the subspace of spinors which can be parameterized by 8 real parameters. This corresponds to $N = 4$ supersymmetry in $d = 3$. It is convenient to combine the spinors into a single complex spinor as follows:

$$\epsilon = \epsilon_R + i\epsilon_L \,. \tag{2.4}$$

Then the above conditions imply:

$$\begin{aligned} -i\epsilon &= \hat{\Gamma}_{0126}\epsilon \\ -\epsilon^* &= \hat{\Gamma}_{3456}\epsilon \\ i\epsilon^* &= \hat{\Gamma}_{6789}\epsilon \,. \end{aligned} \tag{2.5}$$

$*$ denotes complex conjugation.

We will assume that the constraints on the supersymmetry variation parameters obtained in the probe approximation continue to apply in the full solution. These constraints will be used in the next section to obtain supersymmetric solutions for the D3/D5 system following broadly the methods of [8] (see also [9] for the case of strings ending on D-branes).

## 3. Supersymmetric ansatz for the supergravity solution

One approach to solving supergravity equations in situations where some fraction of the supersymmetry is preserved is to solve the Killing spinor equations rather than the full set of non-linear Einstein equations coupled to matter. These Bogomolny type equations tend to be considerably simpler. I will take this approach in what follows.

Type-IIB supergravity has the following massless field content. There are in the NS-NS sector, the dilaton, metric, and NS-NS 2-form. In the R-R sector there is a scalar (also

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known as the axion), a 2-form, and a 4-form with self-dual field strength. We will use the notation of [10] There are two Killing spinor equations in type-IIB supergravity: one for the dilatino and one for the gravitino:

$$\begin{aligned}
\delta\lambda &= \frac{i}{\kappa}\Gamma^M P_M \epsilon^* - \frac{i}{24}\Gamma^{MNP} G_{MNP}\epsilon = 0 \\
\delta\psi_M &= \frac{1}{\kappa}\left(D_M - \frac{i}{2}Q_M\right)\epsilon + \frac{i}{480}\Gamma^{M_1\dots M_5} F_{M_1\dots M_5}\Gamma_M \epsilon + \\
&\quad + \frac{1}{96}\left(\Gamma_M^{NPQ} G_{NPQ} - 9\Gamma^{NP} G_{MNP}\right)\epsilon^* = 0\,. \qquad (3.1)
\end{aligned}$$

The strategy in the remainder of this section will be to develop a sufficiently general ansatz for the supergravity fields to accomodate the D3/D5-brane system. To proceed we first set some notation. We will denote $x^6$ by $y$; indices in the beginning of the Greek alphabet (i.e. $\alpha, \beta, \gamma$ etc) will take values in $3, 4, 5$ (directions transverse to the D3-brane but along the D5-brane); Greek indices in the middle of the alphabet (i.e. $\mu, \nu, \rho$ etc) will take values in $0, 1, 2$ (these are directions shared by the D3- and D5-branes); finally Latin indices in the middle of the alphabet (i.e. $i, j, k$ etc) will take values in $7, 8, 9$ (these directions are transverse to all branes). Hatted indices are flat indices, thus: $G_{\hat{m}\hat{n}\hat{p}} \equiv G_{MNP} e^M_{\hat{m}} e^N_{\hat{n}} e^P_{\hat{p}}$.

We take the Einstein frame metric to be of the following general form:

$$ds_E^2 = H_1^2 \eta_{\mu\nu} dx^\mu dx^\nu + g_{\alpha\beta} dx^\alpha dx^\beta + D^2 \left(dy + A_\alpha dx^\alpha\right)^2 + H_2^2 \delta_{ij} dx^i dx^j\,. \qquad (3.2)$$

In the above ansatz we allow for general off-diagonal terms between coordinates in the $3, 4, 5, 6$ directions. We have collected the off-diagonal terms involving $y$ and the $3, 4, 5$ direction as a vector field in the $3, 4, 5$ directions. In the overall transverse directions we have a diagonal metric. For the solution to be Lorentz invariant in the $0, 1, 2$ directions we take $H_1, H_2, D$ to be independent of these directions. We will set the Ramond-Ramond scalar to zero, thus $\tau = i\tau_2$. In addition we will assume that the only non-zero $G_{\hat{m}\hat{n}\hat{p}}$ are ones with the following index structure: $G_{\hat{3}\hat{4}\hat{5}}, G_{\hat{7}\hat{8}\hat{9}}, G_{\hat{y}\hat{\alpha}\hat{\beta}}, G_{\hat{y}\hat{i}\hat{j}}$. Similarly, for the five-form we will assume that the only non-zero components are those with index structure $F_{\hat{0}\hat{1}\hat{2}\hat{m}\hat{n}}$, where $m, n$ take any value other than $0, 1, 2$. Self-duality of the five-form will, of course, require other components to be non-zero as well.

Inserting the above ansatz into the Killing spinor equations and using the relations (2.5) yields a set of relations between the supergravity fields and $H_1, H_2, D$ and $A_\alpha$ appearing in the metric. Here we present the solution to these constraints. It is convenient to define the combinations $K \equiv H_1^{-2} H_2^2$ and $H \equiv H_1^4 H_2^4$:

$$\begin{aligned}
D^2 &= H^{5/4} K^{-1/2} \\
\tau_2 &= g_s^{-1} H^{1/2} \qquad (3.3) \\
\partial_i A_\alpha &= 0 \\
0 &= \partial_\alpha A_\beta - \partial_\beta A_\alpha - A_\alpha \partial_y A_\beta + A_\beta \partial_y A_\alpha \\
g_{\alpha\beta} &= H^{-3/4} K^{1/2} \delta_{\alpha\beta} \\
\partial_y (A_\beta H) &= \partial_\beta H\,. \qquad (3.4)
\end{aligned}$$

In addition one finds expressions for the self-dual 5-form, and the SL$(2,\mathbb{R})$ invariant 3-form:

$$\begin{aligned} F_5 = & \frac{1}{4\kappa}H\left(\partial_\beta K^{-1} - A_\beta\partial_y K^{-1}\right)dx^0\wedge dx^1\wedge dx^2\wedge dy\wedge dx^\beta + \\ & +\frac{1}{4\kappa}\partial_j\left(HK^{-1}\right)dx^0\wedge dx^1\wedge dx^2\wedge dy\wedge dx^j - \\ & -\frac{1}{8\kappa}\epsilon_{\alpha\beta\delta}\Big(2H^2KA_\alpha\partial_\beta K^{-1}dx^\delta\wedge dy - \\ & \qquad -\{H^2K\left(A_\alpha A\gamma\partial_\gamma K^{-1} - A_\gamma A_\gamma\partial_\alpha K^{-1}\right) + \\ & \qquad\quad +\partial_\alpha K - A_\alpha\partial_y K\}dx^\beta\wedge dx^\delta\Big)\wedge \\ & \wedge dx^7\wedge dx^8\wedge dx^9 + \frac{1}{8\kappa}\epsilon_{ijk}\partial_i(HK^{-1})\times \\ & \times\Big(\left(K^2H^{-2} + KA_\gamma A_\gamma\right)dx^3\wedge dx^4\wedge dx^5 \\ & \quad +\frac{1}{2}KA_\alpha\epsilon_{\alpha\beta\gamma}dy\wedge dx^\beta\wedge dx^\gamma\Big)\wedge dx^j\wedge dx^k\,, \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} G = & \frac{1}{\kappa}\left(-2H^{-9/4}K^{3/2}\partial_y\left(H^{1/2}K^{-1/2}\right) + H^{1/4}A_\gamma\partial_y A_\gamma\right)dx^3\wedge dx^4\wedge dx^5 + \\ & +\frac{1}{2\kappa}H^{1/4}\partial_y A_\delta dy\wedge dx^\alpha\wedge dx^\beta + \\ & +i\left(-\frac{2}{3\kappa}\epsilon_{ijk}\partial_k H^{3/4}dy\wedge x^i\wedge dx^j + \frac{2}{\kappa}H^{-1/4}K^{1/2}\partial_y K^{1/2}dx^7\wedge dx^8\wedge dx^9\right). \end{aligned} \tag{3.6}$$

From these expressions we can extract the string frame metric and the 3-form field strengths in the R-R and NS-NS sectors. They are as follows:

$$\begin{aligned} ds^2 = & K^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + HK^{-1/2}(dy + A_\alpha dx^\alpha)^2 + \\ & +H^{-1}K^{1/2}\delta_{\alpha\beta}dx^\alpha dx^\beta + K^{1/2}\delta_{ij}dx^i dx^j, \\ D_3 = & \frac{1}{\kappa}\left(-\frac{1}{2}\epsilon_{ijk}\partial_k H dy\wedge dx^i\wedge dx^j + \partial_y K dx^7\wedge dx^8\wedge dx^9\right), \\ H_3 = & \frac{1}{\kappa}\Big(-2H^{-5/2}K^{3/2}\partial_y(H^{1/2}K^{-1/2}) + A_\gamma\partial_y A_\gamma)dx^3\wedge dx^4\wedge dx^5 + \\ & \quad +\frac{1}{2}\partial_y A_\delta\epsilon_{\alpha\beta\delta}dy\wedge dx^\alpha\wedge dx^\beta\Big). \end{aligned} \tag{3.7}$$

Some comments are in order here. The off-diagonal components of the metric are completely controlled by the one-form $A$. When $A$ vanishes the metric is completely diagonal. Although the form of the metric and the constraint that $A$ be independent of the $x^i$ coordinates suggests an interpretation of $A$ as a gauge-field, the metric in general will not have a Killing symmetry corresponding to translations in the $y$ direction. This is because in solutions in which the D5-branes are localized in $y$, the putative isometry is explicitly broken. However, below we will consider the case in which the D5-branes are smeared in $y$ producing a Killing symmetry which allows for the interpretation of $A$ as a gauge field. We also note that the NS-NS 3-form field-strength is non-zero, in general, despite the absence of NS-NS 5-branes, this is a feature our solution shares with the Klebanov-Strassler solution and its generalizations.



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The above solutions to the Killing spinor equations guarantee solutions to the Einstein equations. However they have to be supplemented by source equations and Bianchi identities for the 5-form and 3-form field strengths. These equations are:

$$\begin{aligned} dF_5 &= \frac{\kappa}{4} D_3 \wedge H_3\,, \\ dH_3 &= 0\,, \\ dD_3 &= 0\,, \\ d *_E (\tau_2 H_3) &= 4\kappa F_5 \wedge D_3\,, \\ d *_E (\tau_2^{-1} D_3) &= -4\kappa F_5 \wedge H_3\,, \end{aligned} \tag{3.8}$$

where the $*_E$ is the Hodge star with respect to the Einstein metric.

### 3.1 Some checks

To gain some confidence in our solution we perform some elementary checks. We will simply try to reproduce the cases in which one has either D3-branes or D5-branes. These are subcases of our more general set-up in which one of the R-R charges vanishes. The subcase with only D3-branes consists of the following solution of all the constraints:

$$\begin{aligned} H &= 1 \\ K &= (1) + \frac{R_0^4}{(\sum_\alpha x_\alpha^2 + \sum_i x_i^2)^2} \\ A_\alpha &= 0\,. \end{aligned} \tag{3.9}$$

Similarly, the case with only D5-branes is reproduced by the following:

$$\begin{aligned} H &= K = (1) + \frac{R_0^2}{\sum_i x_i^2 + y^2} \\ A &= 0\,. \end{aligned} \tag{3.10}$$

The parentheses around the number 1 in the above equations indicates that they are present in the assymptotically flat cases and absent in the near-horizon ones. Since the D5-brane solution is correctly reproduced an S-duality transformation automatically gives the correct NS5-brane solution as well.

Having convinced ourselves that the known subcases are correctly reproduced, we can proceed to study new cases which have not hitherto been studied in the literature. In the following we will exclusively study the case of intersecting branes, but we emphasize that we believe that the ansatz is general enough to accomodate more general D3/D5-brane webs.

## 4. Intersecting branes

In this section we study the important subcase of a stack of coincident D3-branes intersecting one or more D5-branes. We will set the problem up for localized intersections, but will be forced to shift our attention to the more tractable problem of D5-branes smeared in the $y$ direction.

Consider the case, then, of coinciding $N$ D3-branes located at $x^3 = x^4 = x^5 = x^7 = x^8 = x^9 = 0$. This is a special point in the moduli space of possible configurations. One can separate the D3-branes along $x^3, x^4, x^5$ while continuing to intersect the D5-branes, in addition one can break them along the D5-branes. However, we will only focus on this special point in moduli space where there is an enhanced SO(3) symmetry which rotates the $x^\alpha$ into each other. When describing semi-infinite or finite D3-branes ending on D5-branes, the SO(3) symmetry is broken due to the bending of the D5-branes caused by the tension of the D3-branes. In the special case of infinite D3-brane intersections the pulling of the D5-branes on one side is completely canceled by the D3-branes on the other side, leaving the SO(3) symmetry intact.

Before proceeding to setting up the problem for intersecting branes, we set some notation. We define the usual spherical coordinates in the three dimensional spaces spanned by the $x^\alpha$ and $x^i$. In the former we denote the radial coordinate by $r$ and the metric on the unit sphere by $d\Omega_1^2$, while in the latter we denote the radial direction by $\rho$ and the metric on the unit sphere by $d\Omega_2^2$. We also define the volume form on the two unit spheres by $\omega^1$ and $\omega^2$. Using these definitions, the most general string-frame metric consistent with supersymmetry and the $\mathrm{SO}(3) \times \mathrm{SO}(3)$ symmetry is:

$$\begin{aligned} ds^2 &= K^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + HK^{-1/2}(dy + A_r dr)^2 + \\ &+ H^{-1}K^{1/2}\left(dr^2 + r^2 d\Omega_1^2\right) + K^{1/2}\left(d\rho^2 + \rho^2 d\Omega_2^2\right), \end{aligned} \tag{4.1}$$

provided that $H = H(y, r, \rho)$ and $K = K(y, r, \rho)$. That is, $H$ and $K$ do not depend on the coordinates on the sphere. Notice also that the $\mathrm{SO}(3) \times \mathrm{SO}(3)$ symmetry requires that only the radial component of the one-form $A$ be non-zero. Had there been a Killing vector $\partial/\partial y$, the one-form could be set to zero. However, as stated above this is not the case for D5-branes localized in $y$.

The form-fields can be written as follows:

$$\begin{aligned} F_5 = &\frac{1}{4\kappa}H\left(\partial_r K^{-1} - A_r\partial_y K^{-1}\right)dx^0\wedge dx^1\wedge dx^2\wedge dy\wedge dr + \\ &+\frac{1}{4\kappa}\partial_\rho\left(HK^{-1}\right)dx^0\wedge dx^1\wedge dx^2\wedge dy\wedge d\rho + \\ &+\frac{1}{4\kappa}r^2\rho^2\left(\partial_r K - A_r\partial_y K\right)\omega^1\wedge\omega^2\wedge d\rho \\ &+\frac{1}{4\kappa}r^2\rho^2\partial_\rho\left(HK^{-1}\right)\left(\left(K^2H^{-2} + KA_r^2\right)dr\wedge\omega^1 + KA_r dy\wedge\omega^1\right)\wedge\omega^2, \end{aligned} \tag{4.2}$$

and

$$\begin{aligned} D_3 &= \frac{1}{\kappa}\rho^2\left(-\partial_\rho H dy\wedge\omega^2 + \partial_y K d\rho\wedge\omega^2\right), \\ H_3 &= \frac{1}{\kappa}r^2\left(\left(-2H^{-5/2}K^{3/2}\partial_y\left(H^{1/2}K^{-1/2}\right) + A_\gamma\partial_y A_\gamma\right)dr\wedge\omega^1 + \right. \\ &\left. + \partial_y A_r dy\wedge\omega^1\right). \end{aligned} \tag{4.3}$$

These expressions are somewhat simpler than the general case. The constraint on the one-form $A$ can be expressed as:

$$\begin{aligned} \partial_y (H A_r) &= \partial_r H \\ \partial_\rho A_r &= 0\,. \end{aligned} \tag{4.4}$$

The equations of motion (3.8) are also simpler than the general case. The first equation in (3.8) implies the following relations:

$$\begin{aligned} 0 &= \partial_\rho \partial_y \left( A_r H K^{-1} \right) \\ 0 &= \frac{1}{r^2} \partial_r \left\{ r^2 \left( \partial_r K - A_r \partial_y K \right) \right\} + \frac{1}{\rho^2} \partial_\rho \left\{ \rho^2 \left( \partial_\rho \left( H^{-1} K \right) - A_r^2 K \partial_\rho \left( H K^{-1} \right) \right) \right\} + \\ &\quad + \partial_y K \left\{ H^{-1} \partial_y \left( K H^{-1} \right) + A_r \partial_y A_r \right\} \\ 0 &= \partial_y \partial_r K - A_r \partial_y^2 K - \frac{1}{\rho^2} \partial_\rho \left\{ \rho^2 A_r K \partial_\rho \left( H K^{-1} \right) \right\} \end{aligned} \tag{4.5}$$

$$\begin{aligned} 0 &= \partial_y \left\{ \partial_\rho (H^{-1} K) - \partial_\rho \left( H K^{-1} \right) K A_r^2 \right\} + \frac{1}{r^2} \partial_r \{ r^2 A_r K \partial_\rho (H K^{-1}) \} \\ &\quad - \partial_\rho H \left\{ H^{-1} \partial_y \left( K H^{-1} \right) + A_r \partial_y A_r \right\}. \end{aligned} \tag{4.6}$$

The second equation in the above set of equations is the source equation for the D3-branes and one should have in mind a delta function on the right hand side determining the location of the D3-branes. Similarly, the Bianchi identities for the R-R and NS-NS three-forms impose the following identities:

$$\begin{aligned} 0 &= \partial_\rho \left( \rho^2 \partial_\rho H \right) + \rho^2 \partial_y^2 K \\ 0 &= \partial_\rho \partial_r H \\ 0 &= \partial_r \partial_y K \\ 0 &= \partial_\rho \left\{ H^{-1} \partial_y \left( K H^{-1} \right) \right\} \\ 0 &= \partial_y \left\{ H^{-1} \partial_y \left( K H^{-1} \right) + A_r \partial_y A_r - \frac{1}{r^2} \partial_r \left( r^2 A_r \right) \right\} \end{aligned} \tag{4.7}$$

In the above equations the first equation should really be thought of as a source equation for the D5-branes. The remaining are identities. The last two equation in (3.8) give the following two equations:

$$\begin{aligned} 0 &= \partial_r \left( H \partial_y \ln \left( K H^{-1} \right) \right) + \partial_y \left\{ H \left( \partial_y A_r - A_r \partial_y \ln \left( K H^{-1} \right) \right) \right\} \\ &\quad - H \left( \partial_r K^{-1} - A_r \partial_y K^{-1} \right) \partial_y K \end{aligned} \tag{4.8}$$

$$\begin{aligned} 0 &= -\frac{1}{r^2} \partial_r (r^2 K^{-1} A_r \partial_\rho H) - \partial_\rho (H^{-1} K^{-1} \partial_y K) + \partial_y \left\{ (A_r^2 K^{-1} + H^{-2}) \partial_\rho H \right\} + \\ &\quad + \left\{ H^{-1} \partial_y \left( K H^{-1} \right) + A_r \partial_y A_r \right\} \partial_\rho \left( H K^{-1} \right) \end{aligned} \tag{4.9}$$

The last of these two equations is the equation of motion for the R-R 3-form field strength. Since we have D5-branes in the problem there are magnetic sources for the 3-form. As is usual in such situations one should view this equation as a Bianchi identity rather than as an equation of motion.

The above complex of equations along with appropriate boundary conditions should determine solutions of localized D3/D5-brane intersections. If one simplifies the above equations one finds that (4.8) is satisfied when

$$0 = A_r H K^{-1} \partial_y^2 K\,. \tag{4.10}$$

Thus either $A_r = 0$ or $\partial_y^2 K = 0$. The second option yields delocalized D5-branes as one can see from the source equation for the D5-brane (the first equation in (4.7)). Thus $A_r = 0$ is required.

The final set of equations that need to be satisfied for intersecting D3/D5-branes are relatively simple to state. They consist of the following constraints on $H$ and $K$:

$$\begin{aligned}
0 &= \partial_y \partial_r K \\
0 &= \partial_r H \\
0 &= \partial_y \left(H^{-1} \partial_y \left(K H^{-1}\right)\right) \\
0 &= \partial_\rho \left(H^{-1} \partial_y \left(K H^{-1}\right)\right).
\end{aligned} \tag{4.11}$$

The first two equations in the above are simple to solve. However, the last two are much less trivial when there is no smearing. In fact it is not clear to me that there are non-smeared solutions to the above constraints. Finally there are source equations:

$$\begin{aligned}
\rho_{D3} &= \frac{1}{r^2}\partial_r \left(r^2 \partial_r K\right) + \frac{1}{\rho^2}\partial_\rho \left(\rho^2 \partial_\rho \left(H^{-1} K\right)\right) + \frac{1}{2}\partial_y^2 \left(K H^{-1}\right)^2 \\
\rho_{D5} &= \frac{1}{\rho^2}\partial_\rho \left(\rho^2 \partial_\rho H\right) + \partial_y^2 K\,.
\end{aligned} \tag{4.12}$$

There are two distinct sets of boundary conditions one can impose. The first set are appropriate for assymptotically flat backgrounds. In this case, $K, H \to 1$ as $\rho \to \infty$. Another possibility is to impose the condition that the geometry approach the $\mathrm{AdS}_5 \times \mathrm{S}^5$ near-horizon geometry of the D3-branes as one moves away from the D5-branes in either the $y$ or $\rho$ directions.

### 4.1 Semi-localized solutions

Although the exact equations for a localized solution of intersecting D3/D5-branes were presented above, they are non-linear and, at least at the moment, seem intractable. A considerable simplification occurs if we look for solutions in which the D5-branes are smeared in the $y$ direction. In this case one gains a new isometry with Killing vector $\partial_y$. The functions $K$ and $H$ are then independent of the $y$ coordinate. The constraints (4.11) are satisfied if $H = H(\rho)$ and $K = K(\rho, r)$. The source equations are as follows:

$$\begin{aligned}
\rho_{D5} &= \frac{1}{\rho^2}\partial_\rho \left(\rho^2 \partial_\rho H\right) \\
\rho_{D3} &= \frac{1}{r^2}\partial_r \left(r^2 \partial_r K\right) + \frac{1}{\rho^2}\partial_\rho \left(\rho^2 \partial_\rho \left(H^{-1} K\right)\right)
\end{aligned} \tag{4.13}$$

Another simplification due to the smearing is that the NS-NS 3-form $H_3$ vanishes identically.

Even though this system of equations is much simpler than the one for localized intersections it is not completely trivial. Nevertheless, it is possible to analyze the system in certain assymptotic regimes. To start with it is possible to find the exact solution to the first of the two source equations:

$$H = 1 + \frac{q}{\rho} . \tag{4.14}$$

Although we are imposing boundary conditions appropriate to the zero slope Maldacena limit, the 1 appearing in $H$ cannot be ignored since it is not subdominant. To see this let us try to determine what $q$ is. Since $q$ counts the number of D5-branes, it must be proportional to $n_5$, the number of D5-branes. Secondly, we will consider smearing to be a result of compactifying $y \sim y + 2\pi R$, and then replacing the delta function in the source equation by the inverse volume of $y$. Thus $q \propto R^{-1}$. Finally, we should have $q \propto n_5 \alpha'/R$. In the Maldacena limit we keep $\rho/\alpha'$ fixed, thus the 1 does not become sundominant. Also, as we shall see, to get the appropriate $\mathrm{AdS}_5 \times \mathrm{S}^5$ assymptotic geometry we must retain the 1.

We have not been able to find a closed expression for $K$ by solving the second equation. Nevertheless, we can solve for $K$ in two different assymptotic regimes. The first regime is when we are far away from the D5-branes. This regime is characterized by $q/\rho \ll 1$, thus $H \to 1$. The second source equation then becomes a six-dimensional flat laplacian and the solution for $K$ is simply:

$$K = \frac{R_0}{(r^2 + \rho^2)^2} . \tag{4.15}$$

In other words we have $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry, which is just the assymptotic geometry we wanted. On the other hand we can also solve the equation in the regime in which we are very close to the D5-branes. In this case $q/\rho \gg 1$, and hence $H \to q/\rho$. In this case we can re-write the source equation as:

$$\frac{1}{r^2}\partial_r \left( r^2 \partial_r \left( \frac{\rho K}{q} \right) \right) + \frac{1}{q\rho}\partial_\rho \left( \rho^2 \partial_\rho \left( \frac{\rho K}{q} \right) \right) = 0 . \tag{4.16}$$

In terms of $\eta = 2\sqrt{q\rho}$ the operator acting on $\rho K/q$ in the above equation is:

$$\frac{1}{r^2}\partial_r \left( r^2 \partial_r \right) + \frac{1}{\eta^3}\partial_\eta \left( \eta^3 \partial_\eta \right) . \tag{4.17}$$

This is just the sum of two radial laplacians in 3 and 4 dimensions respectively. A solution for $K$ which only depends on the 7 dimensional radius is given by:

$$K = \frac{qQ}{\rho (r^2 + 4q\rho)^5/2} + \frac{q}{\rho} . \tag{4.18}$$

Where $Q$ is a constant. This solution is valid in the $q/\rho \gg 1$ regime.

Unfortunately we have been unable to make further progress with even the simplified problem of smeared branes. The problem as it stands is well-posed and the differential equation is seperable. Nevertheless new insights seem to be needed to make progress. We leave this to future work.

## 5. Conclusions

In this paper we developed a supersymmetric ansatz for general D3/D5-brane webs. We then focused on the subcase of infinite intersecting D3/D5-branes. In that case we reduced the problem to a set of differential equations involving two unknown functions. These equations while simple to state are non-linear and we were unable to solve them exactly. We could however determine the solution in certain asymptotic regimes in the smeared approximation.

One of our main motivations for undertaking this project was to study $AdS_4$ branes in $AdS_5$ as suggested by [5]. While we were not able to confirm the claims of [5] we have established the general structure of the solution and posed the problem in a simple form. We hope that the results presented in this paper will contribute to a renewed effort to find the full geometry of the D3/D5-brane system. Some progress has already been made in identifying certain features of the system in [11, 12, 13, 14].

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